

STATISTICS AND ADVANCE ALGEBRA POINTERS

The Mean

Among the measures of central tendency, the mean is considered the most popular and most widely used. It is in fact the measure which many people have in mind whenever they talk about averages. It is generally described the center of gravity of a distribution and is, by and large, the most convenient. There are however some limitations on the use of mean as a representative value for a set of measurements.

The Mean of Ungrouped Data

The mean for ungrouped data is computed by simply adding all the values and divided the sum by the number of values. For the sample mean, the formula is

$$\bar{x} = \frac{\sum X}{n}$$

And for the population mean, it is

$$\mu = \frac{\sum X}{N}$$

Example:

The following are the family sizes of a sample of 10 households in a slum area:

2 2 3 3 3 4 4 5 7 7

Find the average family size for these data.

Solution

$$\sum = 2 + 2 + 3 + 3 + 3 + 4 + 4 + 5 + 7 + 7$$

$$\bar{X} = \frac{\sum_{n=10} X}{n} = \frac{40}{10} = 4$$

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Computation of the mean or data where most of the scores occur more than once is facilitated by first multiplying each value by the number of times it appears in the data (frequency) adding the products obtained, and then dividing by n . Examine the following computational procedure for the mean of the preceding example.

X	f	fX
2	2	4
3	3	9
4	2	8
5	1	5
7	2	14
n = 10		Total = 40

$$\bar{X} = \frac{40}{10} = 4$$

If we denote the sum of the values of fX by $\sum fX$, then we have the formula

$$\bar{x} = \frac{\sum X}{n}$$

which yields a value appropriately called the weighted mean. Hence the frequencies serve as weights for the corresponding scores

The Median

The Median is defined as the score point which divides a ranked distribution into two equal parts; it is the value below which lies 50% of the data. Unlike the mean, the median is not sensitive to extreme scores; hence, it is very appropriate when there are values which are relatively large or relatively small compared to most of the scores and in the case of grouped frequency distributions, when open-ended intervals are involved.

The Median of Ungrouped Data

The position of the median of a sample relative to the lowest is generally computed by the formula

$$mdn = \left(\frac{n+1}{2} \right) \text{th score from the lowest.}$$

Similarly for population data,

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$$Mdn = \left(\frac{N+1}{2} \right) \text{th score from the lowest.}$$

Example

Find the median of the following sample data:

6, 8, 15, 18, 23, 24, 42

Solution

$$\begin{aligned} n &= 7 \\ mdn &= \left(\frac{n+1}{2} \right) \text{th score} \\ &= 4^{\text{th}} \text{ score} \\ &= 18 \end{aligned}$$

Find the Median of the following population data:

121, 108, 120, 98, 132, 100, 92, 140, 102, 98

Solution

Arranging the values in ascending order, we obtain

92 98 98 100 102 108 120 121 132 140

Since $N=10$

$$\begin{aligned} Mdn &= \left(\frac{N+1}{2} \right) \text{th score} \\ &= 5.5^{\text{th}} \text{ score} \end{aligned}$$

The median for this set of values is the number halfway between the fifth and the sixth scores. Referring to the array, we found that $X_5 = 103$ and $X_6 = 108$. Therefore

$$Mdn = \left(\frac{102+108}{2} \right) = 105$$

The Mode

The value which we observe to have the highest frequency and which, in certain cases, we use as a measure of central tendency, is known as the mode. While it is possible for a set of values to have no mode because each score appears only once, it is also possible for other sets of values to have more than one mode. Those with two modes are described as *bimodal*, while those with many modes are called *multimodal*. Like the median, the

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mode is not influenced by extreme scores; but unlike the median, it does not indicate anything about the other values in the data like how many are greater and many are less than the most frequently occurring value. Hence, the mode is rarely used.

Example:

Find the mode of the following values;

4, 5, 8, 8, 9, 12, 20

Solution.

The value 8 occurs frequently than any other value; hence, mo = 8.

The Quartiles

The quartiles are the score points which divide a distribution into four equal parts. Twenty – five percent below the first quartile (Q_1), 50% are below the second Quartile (Q_2), and 75% are less than the thirds quartile (Q_3). Note that by definition, the second quartile is equal to the median. Basically, all measures of position related to the median are computed using the same sequence steps. We recall that in determining the median class, we have to multiply n by $\frac{1}{2}$ since 50% are below the median. Logically for first quartile, we multiply n by $\frac{1}{4}$, and for the third quartile, we multiply by $\frac{3}{4}$.

$$Q_1 = X_{LB} + \left(\frac{\frac{n}{4} - cf_p}{f_q} \right) i,$$

where

X_{LB} = lower boundary of the class interval which contains Q_1 (first quartile class)

cf_p = cumulative frequency for the class interval preceding the first quartile class

f_q = frequency in the first quartile class

i = interval size

$$Q_3 = X_{LB} + \left(\frac{\frac{3n}{4} - cf_p}{f_q} \right) i,$$

where X_{LB} , cf_p , f_q are defined by replacing “first quartile class” by “third quartile class” in the above definitions.

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Example:

Find the first and third quartiles of the distribution in table 3.1.

X	f	cf
20-24	2	2
25-29	6	8
30-34	9	17
35-39	10	27
40-44	12	39
45-49	7	46
50-54	4	50
$i = 5$		$n = 59$

Solution

$$\frac{1}{4}n = \frac{1}{4}(50) = 12.5$$

The first quartile class is 30-34

$$Q_1 = X_{LB} + \left(\frac{\frac{n}{4} - cf_p}{f_q} \right) i,$$

$$Q_1 = 29.5 + \left(\frac{12.5 - 8}{9} \right) 5$$

$$Q_1 = 29.5 + 2.5$$

$$Q_1 = 32.00$$

$$\frac{3}{4}n = \frac{3}{4}(50) = 37.5$$

The third quartile class is 40-44

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$$Q_3 = X_{LB} + \left(\frac{\frac{3}{4}n - cf_p}{f_q} \right) i,$$
$$Q_3 = 39.5 + \left(\frac{37.5 - 27}{12} \right) 5$$
$$Q_3 = 39.5 + 4.38$$
$$Q_3 = 43.88$$

The deciles

Nine score points are required to divide a distribution into ten equal parts. They are called deciles and are denoted by $D_1, D_2, D_3, \dots, D_9$. As stressed in the preceding discussion, they are computed in much the same way as how the median and the quartiles calculated. The formulas are as follows:

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$$D_1 = X_{LB} + \left(\frac{\frac{1}{10}n - cf_p}{f_d} \right) i$$

$$D_2 = X_{LB} + \left(\frac{\frac{1}{5}n - cf_p}{f_d} \right) i$$

$$D_3 = X_{LB} + \left(\frac{\frac{3}{10}n - cf_p}{f_d} \right) i$$

$$D_9 = X_{LB} + \left(\frac{\frac{9}{10}n - cf_p}{f_d} \right) i$$

The Percentiles

The percentiles are the ninety-nine score points which divide a distribution into one hundred equal parts. They are generally used to characterize values according to percentage below them. For example, the first percentile (P_1) separates the lowest 1% from the other 99%, the second percentile (P_2) separates the lowest 2% from the other 98% and so on. Fifteen percent are less than the fifteenth percentile and 20% are below the 20th percentile; the middle 80% is determined by the tenth and ninetieth percentiles.

If $k\%$ are less than a given percentile, then

$$P_k = X_{LB} + \frac{\left(\frac{kn}{100} - cf_p \right)}{f_p}$$

where

X_{LB} = lower boundary of the k th percentile class

Cf_p = cumulative frequency for the class interval preceding percentile class

F_p = frequency in the k th percentile class

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Thus, the sixty-fifth percentile is computed by the formula

$$P_{65} = X_{LB} + \frac{\left(\frac{65n}{100} - cf_p\right)}{f_p}$$

Percentile Ranks

Since the various measures of location differ in the terms of percentages of cases below them, it might be interesting to find out how these percentages, called percentile ranks (PR), are computed when percentiles are given. The following can be observed.

1. Percentiles are score-points whereas percentile ranks are percentages. In the statement "40% are less than 90", 90 is the percentile and 40 is the percentile rank. Hence, 90 is the value at the 40th percentile. Similarly, in the statement "70% are greater than 110," the percentile is 110 and the percentile rank is 30; 110 is at the 30th percentile
2. The percentile ranks of Q1, Q2, and Q3 are 25, 50, and 75 respectively.
3. The percentile ranks of D1, D2, D3, ..., D9 are 10, 20, 30, ..., 90 respectively.
4. The percentile rank of P_k is k.

Now, if we replace k in the formula by PR, we have

$$P_{PR} = X_{LB} + \left(\frac{\frac{PR(n)}{100} - cf_p}{f_p}\right) i$$

If the percentile P is given, we determine the required lower boundary X_{LB} by first looking for the class interval which contains P. The frequency in this class interval is substituted for f_p and the cumulative frequency in the preceding class is used for cf_p .

Sequences

A sequence function is any function which has the set of consecutive integers, usually the set of positive consecutive integers beginning with 1, as its domain. When writing a sequence, it is customary to use letter n , instead of x , as the independent variable and a , instead of f , as the name of the function. It is also customary to write a_n (read "a sub n" instead of $a(n)$). For example, $a_n = 2n - 1$ for positive integers n , defines a

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sequence function which contains the ordered pairs (1,1), (2,3), (3,5), (4,7), and so on. The range of such a function, when written in the order a_1, a_2, a_3, a_4 and so on, is called *infinite sequence*. Thus the example above is the infinite sequence 1, 3, 5, 7,

A function whose domain is a finite set of positive integers is called a *finite sequence*. For example, $A_n = 5n$ where $n = 1, 2, 3, 4, 5$ names a finite sequence 5, 10, 15, 20, 25.

The numbers in a sequence, either finite or infinite, are called the *terms* of the sequence. Thus, in a the sequence 1, 3, 5, 7, ... 1 is the first term; 3 is the second term; 5 is the third term; 7, the fourth; and so on. The n th term of the sequence is generally called the *general term* of the sequence. In the sequence above, the general term is $a_n = 2n - 1$.

Example:

Write the first five terms of the infinite sequence

$$a_n = n^2 (2n - 1), n = 1, 2, 3$$

Solution:

$$a_1 = 1^2 [2(1) - 1] = 1(1) = 1$$

$$a_2 = 2^2 [2(2) - 1] = 4(3) = 12$$

$$a_3 = 3^2 [2(3) - 1] = 9(5) = 45$$

$$a_4 = 4^2 [2(4) - 1] = 16(7) = 112$$

$$a_5 = 5^2 [2(5) - 1] = 25(9) = 225$$

An **infinite sequence** is a function whose domain is the entire set of positive integers $\{1, 2, 3, \dots\}$.

A **finite sequence** is a function whose domain is the positive integers from 1 to some fixed integer k , inclusive.

Example:

Find the 30th term of the arithmetic sequence with $a_1 = 3$, and $d = 2$ in the formula for the n th term above, we have

Solution:

$$A_{30} = 3 + (30 - 1)(2) = 3 + 29(2) = 3 + 58 = 61$$

Answer : The 30th term is 61.

In the first three terms of an arithmetic sequence are 3, 7, and 11, what is the 10th term?

Solution:

$$d = 7 - 3 = 4 \text{ or } d = 11 - 7 = 4$$

Substitute $d = 4$, $a_1 = 3$, and $n = 10$ in $a_n = a_1 + (n - 1)d$.

$$a_{10} = 3 + (10 - 1)(4) = 3 + 36 = 39$$

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Answer : The 10th term is 39

Example

If $a_1 = -2$, $a_{12} = 42$, and $n = 12$, find the common difference d and sum of the first twelve terms S_{12} .

Solution:

To find d substitute a_1 , a_{12} , and n in the formula for a_{12}

$$42 = -2 + 11d$$

$$d = 4$$

To find S_{12} , substitute a_1 , a_n , and n in the formula S_{12} .

$$S_{12} = \frac{12}{2}(-2 + 42) = 6(40) = 240$$

Answer: The common difference is 4 and the sum of the first twelve terms is 240.

Sum of the First n Terms of an Arithmetic Sequence

The sum S_n of the first n terms of an arithmetic sequence is

$$S_n = \frac{n}{2}(a_1 + a_n) \text{ or } S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where

a_1 = first term of the sequence

a_n = n th term of the sequence

d = common difference

the n th Term of a Geometric Sequence

If a_1 is the first term of a geometric sequence and r is the common ratio, then the n th term a_n is $a_n = a_1 r^{n-1}$

Find the 8th term of the geometric sequence 64, -32, 16, -8 . . .

Solution:

Using any two consecutive terms, find the common ratio r .

Thus, $16 + (-32) = -1/2$.

$$\begin{aligned} a_8 &= 64 \left(-\frac{1}{2} \right)^7 \\ &= \left(-\frac{1}{128} \right) = -\frac{1}{128} \end{aligned}$$

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Answer: The 8th term is $-\frac{1}{2}$

Find the common ratio r of a geometric sequence whose first term is 2 and whose fourth term is $\frac{1}{4}$.

Solution:

Substitute $a_1 = 2$, $a_4 = \frac{1}{4}$, and $n = 4$ in $a_n = a_1 r^{n-1}$, we have

$$\frac{1}{4} = 2r^3$$

$$\frac{1}{8} = r^3$$

$$r = \frac{1}{2}$$

Answer: The common ratio is $\frac{1}{2}$

Sum of the First n Terms of A Geometric Sequence

If a_1 is the first term, a_n is the last term, and r is the common ratio of geometric sequence, then the sum S_n of the first n terms of the sequence is given by

$$S_n = \frac{a_1(1-r^n)}{1-r} \text{ or } S_n = \frac{a_1 - ra_n}{1-r}, \text{ where } r \neq 1$$

The Fundamental Principle of counting

For a group of k things, if the first can be done independently in n_1 different ways, the second can be done independently in n_2 different ways, "the third can be done independently in n_3 different ways, and so on, until the k th thing, then the total number of ways in which the k things can be done in the stated order is

$$n_1 * n_2 * n_3 * \dots * n_k$$

Example:

In how many ways can three coins fall? Two dice?

Solution

A coin can fall in two ways; therefore three coins can fall in $(2)(2)(2)$ or 8 ways. They are:

Math IV

- 11 -

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HHH TTH
HHT THT
HTH HTT
THH TTT

Similarly, a die can fall in six ways; therefore two dice can fall in $(6)*6$ or 36 ways (see illustration below)

I / II	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Example

Given the digits 0, 2, 5, 6, 9

- How many 3-digit numbers can be formed from these digits if no two digits are to be the same?
- Of the numbers formed in (a) how many are even? How many are odd? How many are greater than 600?
- How many numbers can be formed if a digit may be repeated?

Solution.

- There are four choices for the hundreds digit (excluding zero), four choices for the tens digit (including zero), and only three choices for the units digit. By the Fundamental Principle of Counting, the total number of three-digit numbers under the given condition is

$$(4)(4)(3) = 48$$

- In this situation, a number is even if it ends in 0, 2, 6. The number of three digit numbers ending in either 2 or 6 is

$$(3)(3)(2) = 18$$

and the number of three-digit numbers ending in 0 is

$$(4)(3)(1) = 12$$

hence, there are $18 + 12 = 30$ even numbers

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- (1) A number is odd if it ends in 5 or 9. There are $(3)(3)(2) = 18$ such numbers
- (2) A number is greater than 600 if the hundreds digit is 6 or 9. Thus, $(2)(4)(3) = 24$ of the numbers are greater than 600.
- c. If repetition of digits is allowed, there would be $(4)(5)(5) = 100$ three digit numbers.

Permutations

A permutation is an arrangement of n different objects. The words *rat*, *tar*, *art* are three different permutations for the letters a , r , t . The three permutations for these letters are rta , tra , and atr . There are two different permutations for two different objects. In general, the number of permutations for the n different objects, denoted by ${}_n P_n$ is

$$n(n-1)(n-2)\dots(3)(2)(1) \text{ or } n! \text{ (read "n factorial")}$$

The symbol $n!$ is used to designate the product of all the integers from 1 to n . Thus $1! = 1$, $2! = (2)(1)$, $3! = (3)(2)(1) = 6$, $4! = (4)(3)(2)(1) = 24$, etc. By definition, $0! = 1$. The number of permutations of n objects taken r ($r \leq n$) at a time is denoted by ${}_n P_r$ and is computed by the formula

$${}_n P_r = n(n-1)(n-2)\dots(n-r+1)$$

Example:

In how many ways can 4 boys and 3 girls be seated in a row of 5 chairs?

Solution

$${}_7 P_5 = \frac{7!}{2!} = (7)(6)(5)(4)(3) = 2,520$$

Example:

In how many ways can three of ten students participating in an interschool contest ranked first, second and third?

$${}_{10} P_3 = \frac{10!}{7!} = (10)(9)(8) = 720$$

Example:

How many distinct permutations can be formed from the letters of the word STATISTICS?

Solution

The letters S and T, each appears 3 times, I appears twice, A once and C once. The number of permutations is

$$P = \frac{10!}{3!3!2!} = \frac{(10)(9)(8)(7)(6)(5)(4)}{(3)(2)(2)} = 50,400$$

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Permutations obtained by arranging objects are called **circular permutations**. While there are twenty-four linear permutations for four objects, there are only six distinct circular permutations, each of which is identical to three others. We determine the number of circular permutations by considering one object in a fixed position and calculating for the number of arrangements possible for the remaining three. The number of circular permutations of n different objects is $(n-1)!$

Example:

In how many ways can 8 people be seated at a round table?

Solution

$$P = (8-1)! = 7! = 5,040$$

Combinations

Suppose we have four objects denoted by A, B, C, D. We know from the preceding section that there are twenty-four different permutations of four objects taken three at a time. Thus, if three of the four are chosen in succession, there would be twenty four different sections as shown below.

ABC	ABD	ACD	BCD
ACB	ADB	ADC	BDC
BAC	BAD	CAD	CBD
BCA	BDA	CDA	CDB
CAB	ABD	DAC	DBC
CBA	DBA	DCA	DCB

Without regard to the order, however, there are only four ways in which the three can be chosen from the four. These selections are called *combinations*. The number of combinations of n objects taken r at a time is denoted by ${}_n C_r$. In the present example, we find that for each combination, there are $3!$, or 6, different permutations. Hence, the total number of permutations can be written as

$${}_4 P_3 = {}_4 C_3 * 3!$$

In general, if there are n different objects and we take r at a time,

$${}_n P_r = {}_n C_r * r!$$

Dividing both sides of this equation by $r!$, we obtain

$${}_n C_r = \frac{{}_n P_r}{r!}$$

Therefore, the number of combinations of n objects taken r at a time is

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$${}_n C_r = \frac{n!}{(n-r)!r!}$$

Example:

In how many ways can a committee of 4 be chosen from a group of 8 people?

Solution:

$${}_8 C_4 = \frac{8!}{4!4!} = 70$$

Example:

In how many ways can we select 2 spades and 3 diamonds from a deck of 52 cards?

Solution:

$${}_{13} C_2 = \frac{13!}{11!2!} = 78$$

and the number of ways in which we can select 3 from the 13 diamonds is

$${}_{13} C_3 = \frac{13!}{10!3!} = 286$$

By the Fundamental Principle of Counting, the total number of selections is

$${}_{13} C_2 * {}_{13} C_3 = (78) (286) = 22,308$$

Probability

Types of Probability

There are three different conceptual approaches to the study of probability theory. They are

- The subjective or personalistic approach
- the relative frequency or empirical approach
- the classical or a priori approach

Subjective probability is a personal assessment of the likelihood of occurrence of an event, based on all evidence available. It is appropriate when the person making the judgment finds that there is only one

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opportunity for the event to occur and it will either occur or not occur at that particular instant. A doctor, for example, may use subjective probabilities to assess the chances that a certain patient afflicted with a rare disease will survive an operation. Likewise, an economist may use subjective probabilities to evaluate the likelihood that there will be a recession in the next five years. Without the use of any sophisticated detecting device and relying on past experiences, an expectant mother could only assign a subjective probability value for the sex of her unborn child. You were probably using subjective probabilities when you made statements like "The chances are 60-40 that..." Indeed, the subjective approach is called for when one has very little or no direct evidence regarding the occurrence of an event and he has no choice but to use intuition and certain subjective factors.

Empirical probability is defined as the proportion of times that a particular outcome occurs in a very large number of observations or experiments. It is the relative frequency of an event in past occurrences. If we get the IQ scores of a random sample of 1000 students in our school and find that for every 100 students, one has an IQ score of 150, using the empirical approach to probability theory, we would be justified in saying that the probability that a randomly chosen student of our school has an IQ score of 150 is $\frac{1}{100}$ or 0.01. If for every ten people who enter a department store on a pay-day, three make at least one purchase, the same approach to probability permits us to assert that the probability that a person entering this department store on a pay-day will make at least one purchase is $\frac{3}{10}$ or 0.3.

We define classical probability as follows: If an experiment can result in N equally likely outcomes and an event E can result in n outcomes, then the probability that E will occur is $\frac{n}{N}$. More formally,

$$P(E) = \frac{n(E)}{N},$$

where $n(E)$ = number of sample points on E
 N = total number of sample points in the sample space.

Example:

Two candidates, A and B are running for public office. If the probability the A will win the election is 0.35, what is the probability that B will win?

Solution:

The probability that B will win is equal to the probability that A will lose.

$$P(B) = P(A') = 1 - P(A) = 1 - 0.35 = 0.65$$

Example:

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Find the probability of getting an even number from a single toss of a die.

Solution:

Sample Space = {1, 2, 3, 4, 5, 6}, $N = 6$

Event = {2, 4, 6}, $n(E) = 3$

$P(\text{even number}) = 3/6$ or $1/2$

Example:

If a pair of dice is tossed, find the probability of obtaining a sum of 7.

Solution:

By the Fundamental Principle of Counting, we know that a pair of dice can result in 6×6 or 36 ways ($N = 36$)

The event corresponds to six sample points, namely: (1,6), (6,1), (2,5), (5,2), (3,4), (4,3). Hence,

$P(\text{sum of } 7) = 6/36$ or $1/6$

Example:

A box contains 5 red, 4 blue, and 3 white balls. If a ball is chosen at random, what is the probability that

a. It is not red?

b. It is not white?

Solution:

The box has twelve balls ($N = 12$).

a. $P(\text{not red}) = 1 - P(\text{red}) = 1 - 5/12 = 7/12$

b. $P(\text{not white}) = 1 - P(\text{white}) = 1 - 3/12 = 9/12$ or $3/4$

Mutually exclusive and Nonmutually Exclusive Events

Two events are said to be mutually exclusive if there is no opportunity for them to occur simultaneously if they have no common sample point. "Drawing an ace" and "drawing a king" from a deck of cards are mutually exclusive because it is not possible to obtain an ace which is a king at the same time. Essentially, two or more events are mutually exclusive if their joint probability is zero.

Events that can happen at the same time are nonmutually exclusive events. In particular, they are events which have some sample points in common. The events "drawing an ace" and "drawing a spade," for example, have one sample point in common since one of the four aces is a spade. Hence, these two events are non-mutually exclusive.

The Addition Rule

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Oftentimes, we get interested in the probability of one event or another (or both) occurring in a single experiment. For example, what is the probability of drawing an ace or a king from an ordinary deck? What is the probability of an ace or a spade? If we select a ball at random from a box containing 5 red, 4 blue and 3 white balls, what is the probability that it is either red or blue?

We shall use the symbol " $P(A \text{ or } B)$ " to denote the probability of occurrence of events A or B. Some books, however emphasize the operations use in computing for this probability by using either $P(A \cup B)$ or $P(A + B)$. For the first "card problem" in the preceding paragraph, we observe that there are eight ways in

which we can draw either an ace or a king; therefore, $P(\text{ace or a king})$ is equal to $\frac{8}{52}$ which is also the sum of

the probability of an ace plus the probability of a king. In the case of the "ball problem" we find that since there are three colors selecting a red or a blue ball is equivalent to selecting one which is not white, therefore they have the same probability value.

$$P(\text{not white}) = P(\text{red or blue}) = \frac{9}{7}$$

Now, since $P(\text{red}) = \frac{5}{12}$ and $P(\text{blue}) = \frac{4}{12}$, we have

$$P(\text{red or blue}) = P(\text{red}) + P(\text{blue})$$

It seems reasonable to say that the probability of A or B is equal to the probability of A plus the probability of B. But before we make any generalization, let us consider the situation involving 'ace or spade.' We noted earlier that these two events are not mutually exclusive because they can occur together. One of the four aces is actually among the thirteen spades. We realize that if we merely add $P(\text{ace})$ and $P(\text{spade})$, we would be adding in the probability of the ace of spades twice. In order to correct the probability value resulting from this double counting, we must subtract the joint probability of the two events. Thus we have

$$P(\text{ace or spade}) = P(\text{ace}) + P(\text{spade}) - P(\text{ace and spade})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52} \text{ or } \frac{4}{13}$$

We now make the following addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \text{ for any events A and B.}$$

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If A and B are mutually exclusive events, $P(A \text{ and } B)$ is equal to zero. Therefore the above formula is reduced to

$$P(A \text{ or } B) = P(A) + P(B)$$

Also, if $A_1, A_2, A_3, \dots, A_k$ $P(A_1) + P(A_2) + P(A_3) + \dots + P(A_k)$

Example:

If a card is drawn from an ordinary deck, find the probability of each of the following

- spade or face card
- face card or red card

Solution:

a. $P(S \text{ or } F) = P(S) + P(F) - P(S \text{ and } F)$

$$\begin{aligned} &= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} \\ &= \frac{22}{52} \text{ or } \frac{11}{26} \end{aligned}$$

b. $P(F \text{ or } R) = P(F) + P(R) - P(F \text{ and } R)$

$$\begin{aligned} &= \frac{12}{52} + \frac{56}{52} - \frac{6}{52} \\ &= \frac{32}{52} \text{ or } \frac{8}{13} \end{aligned}$$

Example:

An AB freshman student estimates that the probability that he will pass history 1 is 0.62; the probability that he will pass sociology 1 is 0.50; and the probability that he will pass both subject is 0.40. What is the probability that he will pass at least one of these two subjects?

Solution:

$$\begin{aligned} P(\text{at least one}) &= P(H \text{ or } S) \\ &= P(H) + P(S) - P(H \text{ and } S) \\ &= .62 + .50 - .40 \\ &= .72 \end{aligned}$$

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Conditional Probability

Let us consider once again the “tossing a die” experiment. We let E to denote the event of getting an even number and G the event of getting a number greater than 3. By virtue of the formula, we have

$$P(E) = \frac{3}{6} \text{ and } P(G) = \frac{3}{6}$$

Suppose that after tossing the die, we are told that G has occurred. What is the probability of E? The information we have on the outcome of the experiment essentially reduces the number of possibilities. Whereas before, we had six (1, 2, 3, 4, 5, and 6), now we have only three (4, 5, and 6). Since two of this corresponds to the occurrence of E, we say that the probability of an even number, given a number greater than 3 is $\frac{2}{3}$.

$$P(E/G) = \frac{2}{3}$$

The probability that an event B occurs when it is known that some event A has occurred is called a conditional probability. This is denoted by $P(B/A)$ which is read “the probability of B given A.” The probability of A, given B is written as $P(A/B)$.

From the above example, it can be shown that for any events A and B,

$$P(B/A) = \frac{n(A \text{ and } B)}{n(A)},$$

where $n(A \text{ and } B)$ = number of sample points common to A and B (or number of ways in which A and B can occur together)

$n(A)$ = number of sample points in A.

Note that while $P(A \text{ or } B) = P(B \text{ or } A)$ and $P(A \text{ and } B) = P(B \text{ and } A)$, $P(A/B) \neq P(B/A)$

Example

Determine the probability that the person chosen to head the committee is an educator given that he or she is a business executive.

$$P(B/A) = \frac{n(E \text{ and } B)}{n(B)} = \frac{12}{26} \text{ or } \frac{6}{13}$$

Dividing the numerator and the denominator of the right-hand side of formula by N , we obtain

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$$P(B / A) = \frac{\frac{n(A \text{ and } B)}{N}}{\frac{n(A)}{N}}$$

Since $P(A \text{ and } B) = \frac{n(A \text{ and } B)}{P(A)}$

Dependent and Independent Events

Two events are said to be independent if the occurrence or non-occurrence of one is no effect on the probability of occurrence of the other. When occurrence or non-occurrence of one event does affect the probability of occurrence of the other, the two are said to be dependent on each other. The outcomes associated with tossing a coin and tossing a die are considered independent events because the outcome on the coin does not in any way affect the probability of a number occurring on the die, and the number obtained from the die does not affect the probability of getting either a head or a tail from the coin. Thus, P(H) equals 1/2, no matter what the outcome on the die is. Also, P(1) = 1/6, regardless whether a head or a tail came up from the coin.

This illustration clearly implies that if events A and B are independent,

$P(A/B) = P(A)$

And $P(B/A) = P(B)$

The Multiplication Rule

The multiplication rule, which can be derived by multiplying both of formula by P(A), is employed to evaluate the probability of the simultaneous occurrence of events A and B (also known as the joint probability of A and B). For dependent events, the rule of multiplication is

$P(A \text{ and } B) = P(A) \cdot P(B/A)$.

When A and B are independent events, $P(B?A) = P(B)$; hence for independent events

$P(A \text{ and } B) = P(A) \cdot P(B)$

In general if $A_1, A_2, A_3, \dots, A_k$ are independent events the probability that they will occur together is

$P(A_1, A_2, A_3, \dots, A_k) = P(A_1) \cdot P(A_2) \cdot P(A_3) \dots P(A_k)$

Example

The probability that the raw materials needed by a furniture company will arrive on time is 0.6. If the raw materials arrived on time, the probability that the company will finish the orders on time is 0.8. What is the probability that the raw material arrived on time and the orders re finish on time?

Let A = event that raw materials arrive on time

C = event that the orders are finished on time

$P(A) = 0.6$

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$$\begin{aligned}P(C/A) &= 0.8 \\P(A \text{ and } C) &= P(A) \cdot P(C/A) \\&= (.6) (.8) \\&= .48\end{aligned}$$

Functions

Function is said to be the central idea in the study of mathematics. In every situation, there is always a mathematical function in which one quantity corresponds to another quantity according to some definite rule.

Physical situations and number patterns can be represented by tables, graphs, verbal rules and equations where their interrelationships can be explored.

Function Notation

To denote a functional relationship between two variables x and y , we use the equation

$$y=f(x)$$

which is verbally translated as “ y equals f of x ” or “ y is a function of x .” When we say that y is a function of x , we mean that the value of the variable y depends upon and is uniquely determined by the variable x ; more simply x is the function or rule which allows one to determine the unique value of y , given a value of x . Thus, we say that the variable x is called the independent variable.

Most often, the rule or correspondence is given as an equation in two variables x and y . It is helpful at this point to think of the x -values as inputs and the corresponding y -values as outputs. The function or rule then gives the resulting output from the given input from a given output. For example, the circumference C of a circle depends on the diameter d according to the rule or function $C = \pi D$. For every value of D , a corresponding value for C is obtained by the rule: multiply the value of D by π . In this case, the input is the value of D and the output is the corresponding value of C . In function notation, we say $f(x) = \pi x$, or more specifically $f(D) = \pi D$.

Example:

Given the equation $f(x) = 2x + 3$ or $y = 2x + 3$ where $\leq x \leq 5$, find the $f\left(\frac{1}{2}\right)$ and $f(2)$.

Solution:

The equation $y = 2x + 3$ means that x is multiplied by 2 and the result is added to 3 to obtain y .

The value of the function

$$\text{At } x = \frac{1}{2} \text{ is } y = 2 \cdot \frac{1}{2} + 3 = 4$$

$$\text{At } x = 2 \text{ is } y = 2 \cdot 2 + 3 = 7$$

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Example:

Find $f(x) = x^2 - 2x + 1$ at $f(-2)$; $f(3)$; $f(a)$

Solution:

$$\begin{aligned}F(-2) &= (-2)^2 - 2(-2) + 1 \\ &= 4 + 4 + 1 \\ &= 9\end{aligned}$$

$$f(3) = (3)^2 - 2(3) + 1$$

$$\begin{aligned}f(0) &= (0)^2 - 2(0) + 1 \\ &= 0 - 0 + 1 \\ &= 1\end{aligned}$$

$$\begin{aligned}f(a) &= (a)^2 - 2(a) + 1 \\ &= a^2 - 2a + 1\end{aligned}$$

Operations on Functions

The definitions for operations on functions are as follows.

If f and h are functions

- a. their sum $f + h$ is defined as

$$(\mathbf{f + h})\mathbf{x} = \mathbf{f(x) + h(x)}$$

The domain of $(f + h)$ consists of the number x that are in the domain of f and in the domain of h .

- b. their difference $(f - h)$ is defined as

$$(\mathbf{f - h})\mathbf{x} = \mathbf{f(x) - h(x)}$$

The domain of $(f - h)$ consists of the number x that are in the domain of f and in the domain of h .

- c. their product $f \cdot h$ is defined as

$$(\mathbf{f \cdot h})\mathbf{x} = \mathbf{f(x) \cdot h(x)}$$

The domain of $(f \cdot h)$ consists of the number x that are in the domain of f and in the domain of h .

- d. their quotient $\frac{f}{h}$ is defined as

$$\left(\frac{f}{h}\right)\mathbf{x} = \frac{f(x)}{h(x)}, h(x) \neq 0$$

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The domain of $\frac{f}{h}$ consists of then number x for which $h(x) \neq 0$ that are in the domain of f and in the domain of h .

Example:

Let $f(x) = x+1$ and $h(x) = 2x^2$, find the following and determine the domain in each case.

1. $(f+h)x$

3. $(f \cdot h)x$

2. $(f-h)x$

4. $\left(\frac{f}{h}\right)x$

Solutions:

$$1. (f + h) = f(x) + h(x) = (x+1) + 2x^2 = 2x^2 + x + 1$$

$$2. (f - h)x = f(x) - h(x) = (x + 1) - 2x^2 = -2x^2 + x + 1 \text{ or } 2x^2 - x - 1$$

$$3. (f \cdot h)x = f(x) \cdot h(x) = (x + 1)(2x^2) = 2x^2 + 2x^2$$

$$4. \left(\frac{f}{h}\right)x = \frac{f(x)}{h(x)} = \frac{x+1}{2x^2}$$

Since the operations can be performed on any real number, the domain of f is all real numbers. For he quotient function $\left(\frac{f}{h}\right)$, we must exclude the number 0, because the denominator h has the value 0 when $x = 0$. (Division by 0 is not allowed). Thus the domain of $\frac{f}{h}$ consists of all x for which $x \neq 0$.

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